

Variance Functions:

Constant: $\mathbf{1}$

Power: \mathbf{X}^2

Binomial: $np(1-p)$ where $p = \frac{\mu}{n}$; $V(\mu) = np(1-p)$

Links: initialization of base class returns μ ; p in the logit and subclasses; x elsewhere.

	Link $g(p)$	Inverse $g^{-1}(p)$	Analytic Derivative $g'(p)$
Logit	$z = \log \frac{p}{1-p}$	$p = \frac{e^z}{1+e^z}$	$g'(p) = \frac{1}{p(1-p)}$
Power	$z = x^{\text{pow}}$	$x = z^{\frac{1}{\text{pow}}}$	$g'(x) = \text{pow} \cdot x^{\text{power}-1}$
Inverse	same as above with pow = -1		
Square Root	pow = 0.5		
Identity	pow = 1		
Log	$z = \log x$	$g^{-1}(z) = e^z$	$g'(x) = \frac{1}{x}$
CDFLink/Probit	$z = \Phi^{-1}(p)$	$p = \Phi(z)$	$g'(x) = \frac{1}{\int_{-\infty}^x f(t)dt}$
Cauchy	same as the above with the Cauchy distribution		
CLogLog	$z = \log(-\log p)$	$p = e^{-e^z}$	$g'(p) = -\frac{1}{p \log p}$

Table 1: Link Functions

Initializing the family sets a link property and a variance based on the link(?)

Family	Weights	Deviance	DevResid	Fitted	Predict
Base Class	$\frac{1}{(g'(\mu))^2 \cdot V(\mu)}$	$\frac{\sum_i \text{DevResid}^2}{\text{scale}}$	$(Y - \mu) \cdot \sqrt{\text{weights}}$	$\mu = g^{-1}(\eta)^*$	$\eta = g(\mu)$
Poisson			$\text{sign}(Y - \mu) \sqrt{2Y \log \frac{Y}{\mu} - 2(Y - \mu)}$		
Gaussian			$\frac{(Y - \mu)}{\sqrt{\text{scale} \cdot V(\mu)}}$		
Gamma			Bug?		
Binomial			$\text{sign}(Y - \mu) \sqrt{-2Y \log \frac{\mu}{n} + (n - Y) \log \left(1 - \frac{\mu}{n}\right)}$		
Inverse Gaussian			?		

Table 2: Families

* η is the linear predictor ie., $X\beta$ in the generalized linear model